Math 132 – Calculus II, Fall 2009, Professor: Rakhim Aitbayev

September 16, 2009 **Quiz 2** Name: Solution Key

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1. Find the length of the curve given by the parametric equations $x(t) = e^t \cos t$, $y(t) = e^t \sin t$, $t \in [0, 1]$.

$$L = \int_a^b \sqrt{(x')^2 + (y')^2}\, dt.$$

$$x'(t) = e^t \cos t - e^t \sin t = e^t(\cos t - \sin t)$$

$$y'(t) = e^t \sin t + e^t \cos t = e^t(\sin t + \cos t)$$

$$(x')^2 + (y')^2 = e^{2t}(\cos t - \sin t)^2 + e^{2t}(\sin t + \cos t)^2$$

$$= e^{2t}(\cos^2 t - \cancel{2\sin t \cos t} + \sin^2 t + \sin^2 t + \cancel{2\sin t \cos t} + \cos^2 t)$$

$$= 2e^{2t}$$

$$L = \int_0^1 \sqrt{2e^{2t}}\, dt = \sqrt{2}\int_0^1 e^t\, dt = \sqrt{2}\, e^t\Big|_0^1 = \sqrt{2}(e-1).$$

2. Find the area of the surface generated by revolving about the x-axis the curve

$$y = \sqrt{x+1}, \quad 1 \le x \le 5.$$

$$A = 2\pi \int_a^b y(x)\sqrt{1 + (y'(x))^2}\, dx = 2\pi \int_1^5 \sqrt{x+5}\sqrt{1 + \left(\frac{1}{2\sqrt{x+1}}\right)^2}\, dx$$

$$= \frac{2\pi}{2}\int_1^5 \sqrt{4(x+1)+1}\, dx = \pi \int_1^5 \sqrt{4x+5}\, dx = \left| \begin{array}{l} u = 4x+5 \\ du = 4\,dx \end{array} \right|$$

$$= \frac{\pi}{4}\int_9^{25} u^{1/2}\, du = \frac{\pi}{4}\frac{2}{3} u^{3/2}\Big|_9^{25} = \frac{\pi}{6}\left(25^{3/2} - 9^{3/2}\right)$$

$$= \frac{\pi}{6}(5^3 - 3^3) = \frac{\pi}{6}(125 - 27) = \frac{49\pi}{3}.$$