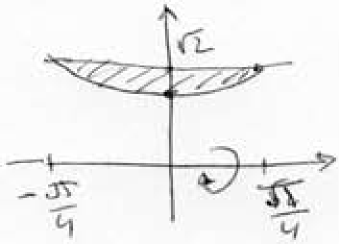
September 4, 2009 **Quiz 1** **Name:** Solution Key

1. Find the volume of the solid generated by revolving about the x-axis the wedge-like plane region bounded by the lines

$$y = \sec x, \quad y = \sqrt{2}, \quad -\frac{\pi}{4} \le x \le \frac{\pi}{4}.$$

$\sec\left(-\frac{\pi}{4}\right) = \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$, Washer Meth

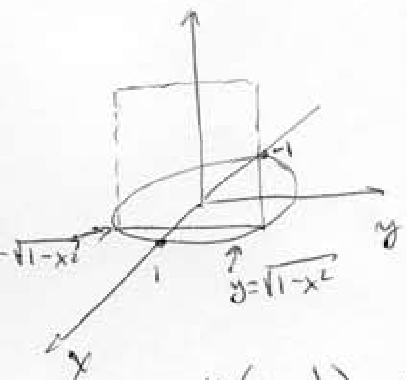


$$V = \pi \int_{-\pi/4}^{\pi/4} \left((\sqrt{2})^2 - (\sec x)^2 \right) dx = \pi \int_{-\pi/4}^{\pi/4} (2 - \sec^2 x)\, dx$$

$$= \pi (2x - \tan x) \Big|_{-\pi/4}^{\pi/4} = \pi \left(2\frac{\pi}{4} - 1 \right) - \pi \left(-2\frac{\pi}{4} + 1 \right)$$

$$= 2\pi \left(\frac{\pi}{2} - 1 \right) = \pi(\pi - 2)$$

2. Find the volume of the solid defined as follows. The base of the solid is the circular disk in the xy-plane described by $x^2 + y^2 = 1$, and the cross-sections by planes perpendicular to the x-axis are squares. Hint: the cross-sections are between $x = -1$ and $x = 1$, and the side of the cross-section square in xy-plane at point $(x, 0)$ is between the curves $y = \sqrt{1 - x^2}$ and $y = -\sqrt{1 - x^2}$.



$$V = \int_{-1}^{1} A(x)\, dx, \quad A(x) - \text{area of cross section}.$$

$$A(x) = \text{side}^2 = \left(\sqrt{1-x^2} - (-\sqrt{1-x^2}) \right)^2 = 4(1 - x^2)$$

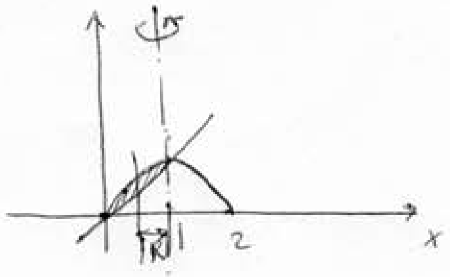
$$V = \int_{-1}^{1} 4(1 - x^2)\, dx = 4\left(x - \frac{x^3}{3} \right) \Big|_{-1}^{1}$$

$$= 4\left(1 - \frac{1}{3}\right) - 4\left(-1 + \frac{1}{3}\right) = 8\left(1 - \frac{1}{3}\right) = 8 \cdot \frac{2}{3} = \frac{16}{3}.$$

3. Find the volume of the solid generated by revolving about the line $x = 1$ the region in xy-plane bounded by the curves $y = 2x - x^2$ and $y = x$.

$y = 2x - x^2 = 0 \Rightarrow x = 0, x = 2$, $y(1) = 2 \cdot 1 - 1^2 = 1$

Shells Method



$R = 1 - x$

$$V = 2\pi \int_0^1 (1 - x)(2x - x^2 - x)\, dx = 2\pi \int_0^1 (1 - x)(x - x^2)\, dx$$

$$= 2\pi \int_0^1 (x - x^2 - x^2 + x^3)\, dx = 2\pi \left(\frac{x^2}{2} - 2\frac{x^3}{3} + \frac{x^4}{4} \right) \Big|_0^1$$

$$= 2\pi \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4} \right) = 2\pi \frac{6 - 8 + 3}{12} = \frac{\pi}{6}.$$