ABSTRACT OF DISSERTATION

Abdulrakhim Aitbayev

The Graduate School
University of Kentucky
1998

ORTHOGONAL SPLINE COLLOCATION FOR NONLINEAR ELLIPTIC BOUNDARY VALUE PROBLEMS

ABSTRACT OF DISSERTATION

A dissertation submitted in partial fulfillment of the requirements of the degree of Doctor of Philosophy at the University of Kentucky

By

Abdulrakhim Aitbayev

Lexington, Kentucky

Director: Bernard Bialecki

Associate Professor of Mathematical and Computer Sciences

Colorado School of Mines

Golden, Colorado

1998

ABSTRACT OF DISSERTATION

ORTHOGONAL SPLINE COLLOCATION FOR NONLINEAR ELLIPTIC BOUNDARY VALUE PROBLEMS

We study the orthogonal spline collocation (OSC) solution of a Dirichlet boundary value problem in a rectangle for a general nonlinear elliptic partial differential equation. The approximate solution is sought in the space of Hermite bicubic splines. In this work, we prove existence and uniqueness of the OSC solution, derive optimal order H^1 and H^2 error estimates, prove the quadratic convergence of Newton's method for solving the OSC problem, prove convergence of the preconditioned conjugate gradient (PCG) method for the solution of a linearized problem at each Newton's iteration, derive a preconditioning technique for the PCG method based on a matrix decomposition algorithm, and present numerical results that support the theoretical analysis.

We prove existence and uniqueness of the OSC solution in a sufficiently small ball with center at a Hermite bicubic spline interpolant of the exact solution of the boundary value problem. The radius of the ball is $\rho = O(|\ln h|^{-(q+2)})$, where $q \geq 0$ is an exponent in the growth conditions on coefficients of the differential operator, and h is a partition parameter. For a special form of the differential operator, we prove that the radius of the uniqueness ball is independent of h. In the general case, we derive optimal order H^1 and H^2 error estimates. For a special form of the differential operator, we derive an optimal H^2 error estimate.

Convergence of Newton's method for solving the OSC problem is proved under the assumption that an initial approximation is in a ball with center at the OSC solution of radius $\rho/2$, where ρ is the radius of a uniqueness ball.

The iterative scheme that we apply for solving the nonlinear algebraic equations can be described as a double stage Newton-PCG method. At each Newton's iteration, we solve, using the PCG method, a linear system with a symmetric positive definite matrix that corresponds to the "normal equation" of the linearized OSC problem. As a preconditioner, we choose a matrix corresponding to a special form of a separable partial differential operator. We show that the convergence rate of the PCG method is independent of h. The numerical results demonstrate the optimal order convergence in L^2, H^1, and H^2 norms. Moreover, the OSC solution exhibits superconvergence properties at the nodal points.

The main conclusions of the dissertation are that the OSC solution of the nonlinear elliptic boundary value problem possesses basically the same convergence properties as the OSC solution of a corresponding linear OSC problem, and that the nonlinear OSC problem can be effectively solved using the Newton-PCG method. This work, to some degree, fills the gap existing at the theoretical level between spline collocation and finite element Galerkin methods for solving nonlinear elliptic boundary value problems.

Abdulrakhim Aitbayev